MATHEMATICAL MODEL OF THE ELECTROCONTACT HEATING OF STEEL BLANKS WITH SQUARE CROSS SECTION

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The present work consists in the elaboration of a mathematical model and the experimental investigation of electrocontact heating of steel blanks. The calculated data are compared with the experiment.

In recent years, electrocontact heating (ECH) of steel blanks for forming or heat treatment has found widespread application because ECH has great operational advantages compared with other kinds of heating [1]. Theoretical investigation of the regimes and parameters of ECH is the first stage in solving the problem of its application in each actual case because the experimental possibilities of investigation are limited. However, the methods of calculating ECH available in the literature either contain rather rough simplifications [2, 3], or they are suitable only for calculating the heating of cylindrical blanks [4]. The present work represents an attempt at working out a mathematical model of ECH of steel blanks with square cross section by alternating current of industrial frequency.

The mathematical model of ECH contains two interconnected problems: a thermal and an electrical problem. Since overflow of heat along the blank may be neglected, the thermal problem reduces to solving the two-dimensional differential equation of thermal conductivity with internal heat sources

$$C\rho\frac{\partial t}{\partial \tau}=\lambda\frac{\partial^2 t}{\partial x^2}+\lambda\frac{\partial^2 t}{\partial y^2}+\frac{\partial \lambda}{\partial x}\frac{\partial t}{\partial x}+\frac{\partial \lambda}{\partial y}\frac{\partial t}{\partial y}+q_V. \tag{1}$$

On the surface of the blank, heat exchange by convection and radiation occurs; the boundary condition has the form

$$-\lambda\frac{\partial t}{\partial n}\Big|_{\mathrm{sur}}=\alpha(t_s-t_c), \tag{2}$$

where

$$\alpha=\alpha_{\text{к}}+\alpha_{\text{r}}. \tag{3}$$

In the case of free convection, α_K is found by the formula [5]

$$\alpha_{\text{к}}=0.47\lambda_{\text{в}}\left(\frac{\beta g}{a\nu}\right)^{1/4}(t_s-t_c)^{1/4}, \tag{4}$$

and for heat exchange by radiation by

$$\alpha_{\text{r}}=C_{\mathrm{rad}}(T_s^4-T_c^4)/(t_s-t_c). \tag{5}$$

To solve Eq. (1), it is indispensable to know the density distribution of the heat sources over the cross section. Since heat is liberated in the blank in consequence of electric current flowing through it, we have

$$q_V=\delta^2/\sigma. \tag{6}$$

If heating is effected by direct current, then the density of the heat sources over the cross section of the blank is the same at all points and equal to

$$q_V=(U/l)^2\sigma. \tag{7}$$

In the case of heating by alternating current, the distribution of the current, and together with it of q_V, because of the surface effect, becomes very nonuniform.

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The electrical problem consists in finding the distribution $\delta(x, y)$ over the cross section of a steel blank. i.e., it reduces to solving the differential equation

$$\frac{\partial^2\delta}{\partial x^2}+\frac{\partial^2\delta}{\partial y^2}=\frac{1}{\mu}\frac{\partial\mu}{\partial x}\frac{\partial\delta}{\partial x}+\frac{1}{\mu}\frac{\partial\mu}{\partial y}\frac{\partial\delta}{\partial y}+j\omega\sigma\mu\delta. \tag{8}$$

This equation is obtained from Maxwell's equation by a method described in [6]. If $\partial\mu/\partial x = \partial\mu/\partial y \equiv 0$ (i.e., the steel is nonmagnetic), then (8) coincides with the equation presented in [6].

It is expedient to seek the solution of (8) in the form

$$\delta(x, y)=\delta_0\gamma(x, y). \tag{9}$$

In this case, the function $\gamma(x, y)$ is the solution of Eq. (8), and on the surface of the blank it assumes values equal to unity. With such an approach it is assumed that the current density on the perimeter of the cross section of the blank is the same and equal to δ_0. Though it is known that ac is forced into the corners of the conductor (see, e.g., [7-9]), Eq. (9) can nevertheless be considered correct for steel blanks in the first approximation because through the closing of the magnetic flux inside a steel conductor [10], the squeezing of the current into the corners is considerably weakened.

The current density on the surface of the blank can be found from the values of the full current flowing through the blank and is equal to

$$I=4\int_0^{a/2}\int_0^{a/2}\delta_0\gamma(x, y)\,dxdy. \tag{10}$$

On account of the symmetry, integration here is done over a quarter cross section. On the other hand,

$$I=U/(R^2+X^2)^{1/2}. \tag{11}$$

Consequently, to calculate the full current, we have to know the resistance and the reactance of the heated blank. The surface effect is the reason that the resistance and reactance are larger than they would be if it were disregarded.

The reactance consists of two parts:

$$X=X_g+X_s \tag{12}$$

The contribution of X_g is determined by the geometry of the part, and for a straight blank with square cross section it is [11]

$$X_g=2\omega l\left(\ln\frac{4.48l}{a}-1\right)\cdot 10^{-7}. \tag{13}$$

The resistance and reactance of the blank due to the surface effect can be calculated by using Neiman's method [12]. With this method, the following dimensionless parameter is examined:

$$Q=\frac{a}{4}(\omega\mu_0\mu_{rel}\sigma)^{1/2}. \tag{14}$$

Now

$$R=\begin{cases}(1+0.55Q^2-0.025Q^4)R_0, & \text{if } Q\leqslant 2,\\ 1.4QR_0, & \text{if } Q>2,\end{cases} \tag{15}$$

$$X_s=\begin{cases}(1.26Q^2-0.42Q^4)R_0, & \text{if } Q\leqslant 1,\\ 0.84QR_0, & \text{if } Q>1.\end{cases} \tag{16}$$

Equation (14) contains an unknown parameter: the relative magnetic permeability of steel depending on the magnetic field intensity, current intensity, and the temperature. Consequently, to calculate R and X_s, we have to find μ_{rel}.

A conductor with a current flowing through it induces around itself a magnetic field whose intensity can be found in the first approximation by the formula

$$H=I/4a. \tag{17}$$

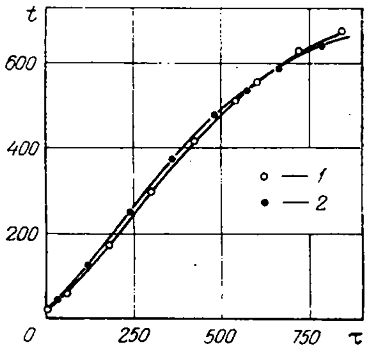

Fig. 1. Experimental and theoretical graphs of heating blanks of steel 08KP (solid lines: calculation, t, °C; τ, sec): 1) center of the cross section of the blank; 2) fin.

When I and H are large, the relative magnetic permeability of steel is practically independent of its composition and is determined only by the magnitude of H [13]:

$$\mu^*_{rel} = 1.32 \cdot 10^7 / H^{1.21}. \tag{18}$$

Here, the asterisk * denotes that the magnetic permeability is calculated at low temperature (close to room temperature).

To take into account the decrease of μ_{rel} in heating, we have to write

$$\mu_{rel} = 1 + (\mu^*_{rel} - 1)\,\xi(t). \tag{19}$$

The dimensionless function $\xi(t)$, which takes the dependence of μ_{rel} of steel on the temperature into account, can be approximated with good accuracy by the expression

$$\xi(t) = \left(1 - \left(\frac{t}{t_C}\right)^4\right)^{1/2}, \tag{20}$$

where t_C is the Curie point. Above the Curie point, $\xi(t) \equiv 0$. For nonmagnetic steel, μ_{rel} = 1 and is independent of the temperature.

It can easily be seen that Eqs. (11), (14)-(20) are interrelated and can be solved by the iteration method. Besides that, (1) and (8) do not have an analytical solution. Therefore the above-described model of ECH can be realized only with the aid of a computer.

The adequacy of the suggested model was verified by comparing experimental and theoretical graphs of heating an experimental blank. The calculations were carried out by the explicit finite-difference method of first order of accuracy [14] on a Minsk-22 computer.

On an experimental ECH installation described in [15], a blank of steel 08KP, 50 × 50 × 300 mm in size, was heated. With the aid of Chromel—Alumel thermocouples and a multipoint electronic potentiometer, the temperatures of the following three representative points were measured and recorded during the heating process: the center of the blank, the center of the face, and the center of a fin.

To eliminate heat removal through the thermocouple electrodes, the thermocouples measuring the temperatures of the face and the fin were inserted into the blank; for this, channels of 3 mm diam. were drilled, and in them the thermocouples were mounted in a two-channel porcelain duct. Since it is fairly difficult to drill three channels in one cross section, and in order to reduce distortions in the current distribution over the cross section of the blank, the channels were drilled in cross sections 5 mm from each other.

The agreement between the experimental and the theoretical graphs of heating is very good (Fig. 1). The divergence between the measured and the calculated temperatures do not exceed 2 or 3%. Experiments as well as calculation showed that at the first stage of heating, when heat losses are slight, the surface of the blank is hotter than the center, and the fin is hottest of all. This involves a fairly large temperature gradient between the fin and the center of the cross section of the blank which attains as much as 14°C, regardless of the fact that the dimensions of the cross section are small and the heating rates are relatively low.

Upon further increase in temperature, the heat losses, especially through the fin, increase, and the temperature of the fin is eventually lower than the temperature at any other point of the cross section of the blank.

The voltage in the blank during the experiment was 1.40 V, and in the calculation 1.32 V was taken. The difference is due to the losses in the metal structure of the experimental installation, which caused an increased voltage drop in the blank.

Thus it may be concluded that the suggested mathematical model of ECH describes the process of ECH of a steel blank of square cross section quantitatively, and not only qualitatively, correctly, and that it may also be used for calculating the principal parameters of ECH. This model can also be easily generalized for the case of a blank with rectangular cross section.

NOTATION

a, side of the square cross section of the blank; l, length of the blank; C, thermal capacity; ρ, density; λ, thermal conductivity; σ, electrical conductivity; $\mu = \mu_0 \mu_{rel}$, absolute magnetic permeability of steel; μ_0, magnetic permeability of vacuum; μ_{rel}, relative magnetic permeability of steel; U, voltage in the blank; I, total current; ω, angular frequency of the ac; δ, current density; δ_0, current density on the surface of the blank; t, temperature; T = t + 273°C, absolute temperature; t_s, temperature of the surface of the blank; t_c, ambient temperature; τ, time, q_V, volumetric density of the internal heat sources; $\partial t/\partial n|_{sur}$, gradient of the temperature field on the surface of the blank; α, total heat-transfer coefficient; α_K, convective heat-transfer coefficient; α_r, radiative heat-transfer coefficient; λ_B, thermal conductivity of air; β, coefficient of thermal expansion; ν, kinematic viscosity of air, g, free gravitational acceleration; C_{rad}, reduced radiation coefficient; R, resistance; X, reactance of the blank; R_0, resistance of the blank to dc; X_s, reactance of the blank due to surface effect; H, magnetic field intensity.

LITERATURE CITED

1. H. G. Heine, Iron Steel Eng., 54, No. 2 (1977).
2. D. I. Romanov, Electrocontact Heating of Metals [in Russian], Mashinostroenie, Moscow (1965).
3. G. S. Kovrev, Electrocontact Heating in the Processing of Nonferrous Metals [in Russian], Metallurgiya, Moscow (1975).
4. A. Kh. Valeev, in: Electrocontact Heating of Steel Blanks [in Russian], Informstandart-élektro, Moscow (1968).
5. M. A. Mikheev and I. M. Mikheev, Fundamentals of Heat Transfer [in Russian], 2nd ed., Énergiya, Moscow (1977).
6. S. D. Kupalyan, Transactions of the Ordzhonikidze MAI [in Russian], Issue 1966, Oborongiz (1956).
7. H. C. Forbes and L. I. Gorman, Trans. AIEE, 52, 516 (1933).
8. J. Daley, Trans. AIEE, 58, 687 (1939).
9. Yu. L. Mukoseev, Distribution of Alternating Current in Conductors [in Russian], GÉI, Moscow (1959).
10. L. P. Neiman and P. L. Kalantarov, Theoretical Foundations of Electrical Engineering [in Russian], Part III, GÉI, Moscow (1959).
11. B. M. Strunskii, Short Networks of Electric Furnaces [in Russian], Metallurgizdat, Moscow (1962).
12. L. P. Neiman, Surface Effect in Ferromagnetic Bodies [in Russian], GÉI, Leningrad (1949).
13. W. Ullrich, "Elektrische Erwarmungsverfachren in Walzwerken," Blech, No. 2 (1966).
14. D. Potter, Computational Physics, Wiley (1973).
15. V. Z. Remnyakov, Yu. P. Shalaev, and A. G. Rogovskii, Metallurgical Heat Engineering. Branch Collection No. 6 [in Russian], Metallurgiya, Moscow (1978).